

WHAT IS CLAIMED IS:

1 1. A method performed by a custodian to share a secret S among n secret
2 owners, the method comprising the steps of:
3 choosing two large primes P and Q ;
4 computing a product $N = PQ$;
5 computing a product $M = (P-1)(Q-1)$;
6 choosing n random numbers q_1 through q_n that are relatively prime to M ;
7 determining a number d such that a product of q_1 through q_n and $d \bmod M$
8 equals one;
9 computing S^d ;
10 distributing n secret owner pieces to each of the n secret owners, wherein each
11 of the secret owner pieces includes S^d and one of the numbers q_1 through q_n ; and
12 deleting the secret S , P , Q , M , q_1 through q_n , and d .

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4 computing $S^q \bmod N$, where q represents the one of the numbers q_1 through
5 q_n contained in another of the n secret owner pieces; and replacing S' with $S^q \bmod N$.

1 5. A method performed by a custodian to share a secret S among n secret
2 owners, the method comprising the steps of:

3 choosing two large primes P and Q ;
4 computing a product $N = PQ$;
5 computing a product $M = (P-1)(Q-1)$;
6 choosing $n+1$ random numbers q_1 through q_n and d' that are relatively prime
7 to M ;
8 determining a number d such that a product of q_1 through q_n , d' , and $d \bmod M$
9 equals one;
10 computing S^d ;
11 distributing n secret owner pieces to each of the n secret owners, wherein each
12 of the secret owner pieces includes S^d and one of the numbers q_1 through q_n ; and
13 deleting the secret S , P , Q , M , q_1 through q_n , and d .

1 6. A method as in claim 5, the method further comprising the steps of:
2 receiving a first of the n secret owner pieces from one of the n secret
3 owners; and
4 computing and storing $S' = S^{dq} \bmod N$, where q represents the one of the
5 numbers q_1 through q_n contained in the first of the n secret owner pieces.

1 7. A method as in claim 6, the method further comprising the steps of:
2 receiving a second of the n secret owner pieces from another one of the n
3 secret owners;
4 computing $S^q \bmod N$, where q represents the one of the numbers q_1 through
5 q_n contained in the second of the n secret owner pieces; and
6 replacing S' with $S^q \bmod N$.

1 8. A method as in claim 7, further comprising the step of:
2 each time another of the secret owner pieces is received from another one of
3 the n secret owners;
4 computing $S^q \bmod N$, where q represents the one of the numbers q_1 through
5 q_n contained in the another of the n secret owner pieces; and
6 replacing S' with $S^q \bmod N$.

1 9. A method as in claim 8, further comprising the steps of:
2 after all n secret owner pieces has been received;
3 computing $S^{d'}$ mod N ; and
4 replacing S' with $S^{d'}$ mod N .

1 10. A method performed by a custodian to share a secret S among n secret
2 owners such that any k of the n secret owners may reconstruct the secret, the method
3 comprising the steps of:

4 choosing two large primes P and Q , such that PQ is greater than S ;
5 computing and storing a product $N = PQ$;
6 computing and storing a product $M = (P-1)(Q-1)$;
7 choosing n random numbers e_1 through e_n that are relatively prime to N ;
8 choosing another random number e that is relatively prime to N ;
9 choosing n numbers d_1 through d_n such that $e_i d_i \bmod M$ equals one for
10 $1 \leq i \leq n$;
11 choosing another number d such that $ed \bmod M$ is equal to one;
12 generating and storing a database of $\binom{n}{k}$ values, where each value is the
13 product of d and a unique k of the d_i numbers for $1 \leq i \leq n$;
14 deleting P , Q , and M ;
15 computing S^e ;
16 distributing n secret owner pieces to each of the n secret owners, wherein each
17 of the secret owner pieces includes S^e and one of the numbers e_1 through e_n ; and
18 deleting the secret S and e_1 through e_n , e , d_1 through d_n , and d .

11. A method as in claim 10, the method further comprising the steps of:
 receiving a first of the n secret owner pieces from one of the n secret owners;
 and
 computing and storing $S' = S^{e_f} \bmod N$, where f represents the one of the
 numbers e_1 through e_n contained in the first of the n secret owner pieces.

12. A method as in claim 11, the method further comprising the steps of:
 receiving a second of the n secret owner pieces from another one of the n
 secret owners;
 computing $S'^q \bmod N$, where q represents the one of the numbers e_1 through
 e_n contained in the second of the n secret owner pieces; and replacing S' with $S'^q \bmod N$.

13. A method as in claim 12, further comprising the step of:
 each time another of the secret owner pieces is received from another one of
 the n secret owners;
 computing $S'^q \bmod N$, where q represents the one of the numbers e_1 through
 e_n contained in the another of the n secret owner pieces; and replacing S' with $S'^q \bmod N$.

14. A method as in claim 13, further comprising the steps of:
 after k secret owner pieces have been received,
 retrieving from the database a value c from among the $\binom{n}{k}$ values, wherein the
 value c corresponds to the k secret owner pieces that were received by the custodian;
 computing $S'^c \bmod N$; and
 replacing S' with $S'^c \bmod N$.

15. A method performed by a custodian to share a secret S among n secret
 owners such that any k of the n secret owners may reconstruct the secret, the method
 comprising the steps of:

choosing two large primes P and Q , such that PQ is greater than S ;
 computing and storing a product $N = PQ$;
 computing and storing a product $M = (P-1)(Q-1)$;
 choosing n random numbers e_1 through e_n that are relatively prime to N ;
 choosing random numbers e and e' that are relatively prime to N ;
 choosing n numbers d_1 through d_n such that $e_i d_i \bmod M$ equals one for
 $1 \leq i \leq n$;
 choosing numbers d and d' such that $ed \bmod M$ is equal to one and such that
 $e'd' \bmod M$ is equal to one;
 generating and storing a database of $\binom{n}{k}$ values, where each value is the
 product of d and a unique k of the d_i numbers for $1 \leq i \leq n$;
 deleting P , Q , and M ;
 computing $S^{ee'}$;
 distributing n secret owner pieces to each of the n secret owners, wherein each
 of the secret owner pieces includes $S^{ee'}$ and one of the numbers e_1 through e_n ; and
 deleting the secret S and e_1 through e_n , e , d_1 through d_n , and d .

16. A method as in claim 15, the method further comprising the steps of:
 receiving a first of the n secret owner pieces from one of the n secret owners;
 and
 computing and storing $S' = S^{ee'f} \bmod N$, where f represents the one of the
 numbers e_1 through e_n contained in the first of the n secret owner pieces.

17. A method as in claim 16, the method further comprising the steps of:
 receiving a second of the n secret owner pieces from another one of the n
 secret owners;
 computing $S^q \bmod N$, where q represents the one of the numbers e_1 through
 e_n contained in the second of the n secret owner pieces; and replacing S' with $S^q \bmod N$.

18. A method as in claim 17, further comprising the step of:

2 each time another of the secret owner pieces is received from another one of
3 the n secret owners;

4 computing $S^q \bmod N$, where q represents the one of the numbers e_1 through
5 e_n contained in the another of the n secret owner pieces; and replacing S' with $S^q \bmod N$.

1 19. A method as in claim 18, further comprising the steps of:
2 after k secret owner pieces have been received,

3 retrieving from the database a value c from among the $\binom{n}{k}$ values, wherein the
4 value c corresponds to the k secret owner pieces that were received by the custodian;
5 computing $S^c \bmod N$;
6 replacing S' with $S^c \bmod N$;
7 computing $S^{d'} \bmod N$; and
8 replacing S' with $S^{d'} \bmod N$.

1 20. A method performed by a custodian to share a secret among n secret
2 owners such that any k of the n secret owners may reconstruct the secret, the method
3 comprising the steps of:

4 encrypting the secret so as to generate an encrypted secret;
5 deleting the secret; and
6 performing a forward k out of n secret sharing algorithm on the encrypted
7 secret so as to generate n secret owner pieces.

1 21. A method as in claim 20, further comprising the step of:
2 distributing the n secret owner pieces to the n secret owners.

1 22. A method as in claim 21, further comprising the step of:
2 receiving k secret owner pieces from k secret owners.

1 23. A method as in claim 22, further comprising the step of:

performing a reverse k out of n secret sharing algorithm on the k secret owner pieces so as to recreate the encrypted secret.

24. A method as in claim 23, further comprising the step of:
decrypting the encrypted secret so as to recreate the secret.

25. A method as in claim 20, wherein the step of performing a forward k out of n secret sharing algorithm includes the steps of:
dividing the encrypted secret into k pieces; and
performing n polynomial evaluations at n points of a degree- k polynomial using the k pieces of the encrypted secret as polynomial coefficients;
wherein each of the k secret owner pieces includes a result of one of the n polynomial evaluations and a corresponding one of the n points.

26. A method as in claim 25, further comprising the steps of:
distributing the n secret owner pieces to the n secret owners;
receiving k secret owner pieces from k secret owners; and
performing a reverse k out of n secret sharing algorithm on the k secret owner pieces so as to recreate the encrypted secret; wherein the step of performing a reverse k out of n secret sharing algorithm includes the steps of generating a system of k linear equations and solving the system of k linear equations for the k pieces of the encrypted secret.

27. A method as in claim 26, further comprising the step of:
assembling the k pieces of the encrypted secret so as to recreate the encrypted secret; and
decrypting the encrypted secret so as to recreate the secret.

28. A computer readable storage medium having embodied thereon computer readable program code suitable for programming a computer to perform a method performed by a custodian to share a secret S among n secret owners, the method comprising the steps of:

5 choosing two large primes P and Q ;
 6 computing a product $N = PQ$;
 7 computing a product $M = (P-1)(Q-1)$;
 8 choosing n random numbers q_1 through q_n that are relatively prime to M ;
 9 determining a number d such that a product of q_1 through q_n and $d \bmod M$
 10 equals one;
 11 computing S^d ;
 12 distributing n secret owner pieces to each of the n secret owners, wherein each
 13 of the secret owner pieces includes S^d and one of the numbers q_1 through q_n ; and
 14 deleting the secret S , P , Q , M , q_1 through q_n , and d .

1 29. A computer readable storage medium having embodied thereon
 2 computer readable program code suitable for programming a computer to perform a method
 3 performed by a custodian to share a secret S among n secret owners, the method comprising
 4 the steps of:
 5 choosing two large primes P and Q ;
 6 computing a product $N = PQ$;
 7 computing a product $M = (P-1)(Q-1)$;
 8 choosing $n+1$ random numbers q_1 through q_n and d' that are relatively prime to
 9 M ;
 10 determining a number d such that a product of q_1 through q_n , d' , and $d \bmod M$
 11 equals one;
 12 computing S^d ;
 13 distributing n secret owner pieces to each of the n secret owners, wherein each
 14 of the secret owner pieces includes S^d and one of the numbers q_1 through q_n ; and
 15 deleting the secret S , P , Q , M , q_1 through q_n , and d .

1 30. A computer readable storage medium having embodied thereon
 2 computer readable program code suitable for programming a computer to perform a method
 3 performed by a custodian to share a secret S among n secret owners such that any k of the n
 4 secret owners may reconstruct the secret, the method comprising the steps of:

5 choosing two large primes P and Q , such that PQ is greater than S ;
 6 computing and storing a product $N = PQ$;
 7 computing and storing a product $M = (P-1)(Q-1)$;
 8 choosing n random numbers e_1 through e_n that are relatively prime to N ;
 9 choosing another random number e that is relatively prime to N ;
 10 choosing n numbers d_1 through d_n such that $e_i d_i \bmod M$ equals one for
 11 $1 \leq i \leq n$;
 12 choosing another number d such that $ed \bmod M$ is equal to one;
 13 generating and storing a database of $\binom{n}{k}$ values, where each value is the
 14 product of d and a unique k of the d_i numbers for $1 \leq i \leq n$;
 15 deleting P , Q , and M ;
 16 computing S^e ;
 17 distributing n secret owner pieces to each of the n secret owners, wherein each
 18 of the secret owner pieces includes S^e and one of the numbers e_1 through e_n ; and
 19 deleting the secret S and e_1 through e_n , e , d_1 through d_n , and d .

31. A computer readable storage medium having embodied thereon
 computer readable program code suitable for programming a computer to perform a method
 performed by a custodian to share a secret S among n secret owners such that any k of the n
 secret owners may reconstruct the secret, the method comprising the steps of:

5 choosing two large primes P and Q , such that PQ is greater than S ;
 6 computing and storing a product $N = PQ$;
 7 computing and storing a product $M = (P-1)(Q-1)$;
 8 choosing n random numbers e_1 through e_n that are relatively prime to N ;
 9 choosing random numbers e and e' that are relatively prime to N ;
 10 choosing n numbers d_1 through d_n such that $e_i d_i \bmod M$ equals one for
 11 $1 \leq i \leq n$;
 12 choosing numbers d and d' such that $ed \bmod M$ is equal to one and such that
 13 $e'd' \bmod M$ is equal to one;

14 generating and storing a database of $\binom{n}{k}$ values, where each value is the
 15 product of d and a unique k of the d_i numbers for $1 \leq i \leq n$;
 16 deleting P , Q , and M ;
 17 computing S^{ee} ;
 18 distributing n secret owner pieces to each of the n secret owners, wherein each
 19 of the secret owner pieces includes S^{ee} and one of the numbers e_1 through e_n ; and
 20 deleting the secret S and e_1 through e_n , d_1 through d_n , and d .

1 32. A computer readable storage medium having embodied thereon
 2 computer readable program code suitable for programming a computer to perform a method
 3 performed by a custodian to share a secret among n secret owners such that any k of the n
 4 secret owners may reconstruct the secret, the method comprising the steps of:
 5 encrypting the secret so as to generate an encrypted secret;
 6 deleting the secret; and
 7 performing a forward k out of n secret sharing algorithm on the encrypted
 8 secret so as to generate n secret owner pieces.

1 33. A computer comprising a processor and a computer readable storage
 2 medium coupled to the processor having embodied thereon processor readable program code
 3 suitable for programming the computer to perform a method performed by a custodian to
 4 share a secret S among n secret owners, the method comprising the steps of:
 5 choosing two large primes P and Q ;
 6 computing a product $N = PQ$;
 7 computing a product $M = (P-1)(Q-1)$;
 8 choosing n random numbers q_1 through q_n that are relatively prime to M ;
 9 determining a number d such that a product of q_1 through q_n and $d \bmod M$
 10 equals one;
 11 computing S^d ;
 12 distributing n secret owner pieces to each of the n secret owners, wherein each
 13 of the secret owner pieces includes S^d and one of the numbers q_1 through q_n ; and

14 deleting the secret S , P , Q , M , q_1 through q_n , and d .

1 34. A computer comprising a processor and a computer readable storage
2 medium coupled to the processor having embodied thereon processor readable program code
3 suitable for programming a computer to perform a method performed by a custodian to share
4 a secret S among n secret owners, the method comprising the steps of:
5 choosing two large primes P and Q ;
6 computing a product $N = PQ$;
7 computing a product $M = (P-1)(Q-1)$;
8 choosing $n+1$ random numbers q_1 through q_n and d' that are relatively prime to
9 M ;
10 determining a number d such that a product of q_1 through q_n , d' , and $d \bmod M$
11 equals one;
12 computing S^d ;
13 distributing n secret owner pieces to each of the n secret owners, wherein each
14 of the secret owner pieces includes S^d and one of the numbers q_1 through q_n ; and
15 deleting the secret S , P , Q , M , q_1 through q_n , and d .

16 35. A computer comprising a processor and a computer readable storage
17 medium coupled to the processor having embodied thereon processor readable program code
18 suitable for programming a computer to perform a method performed by a custodian to share
19 a secret S among n secret owners such that any k of the n secret owners may reconstruct the
20 secret, the method comprising the steps of:
21 choosing two large primes P and Q , such that PQ is greater than S ;
22 computing and storing a product $N = PQ$;
23 computing and storing a product $M = (P-1)(Q-1)$;
24 choosing n random numbers e_1 through e_n that are relatively prime to N ;
25 choosing another random number e that is relatively prime to N ;
26 choosing n numbers d_1 through d_n such that $e_i d_i \bmod M$ equals one for
27 $1 \leq i \leq n$;
28 choosing another number d such that $ed \bmod M$ is equal to one;

14 generating and storing a database of $\binom{n}{k}$ values, where each value is the
 15 product of d and a unique k of the d_i numbers for $1 \leq i \leq n$;
 16 deleting P , Q , and M ;
 17 computing S^e ;
 18 distributing n secret owner pieces to each of the n secret owners, wherein each
 19 of the secret owner pieces includes S^e and one of the numbers e_1 through e_n ; and
 20 deleting the secret S and e_1 through e_n , e , d_1 through d_n , and d .

1 36. A computer comprising a processor and a computer readable storage
 2 medium coupled to the processor having embodied thereon processor readable program code
 3 suitable for programming the computer to perform a method performed by a custodian to
 4 share a secret S among n secret owners such that any k of the n secret owners may reconstruct
 5 the secret, the method comprising the steps of:

6 choosing two large primes P and Q , such that PQ is greater than S ;
 7 computing and storing a product $N = PQ$;
 8 computing and storing a product $M = (P-1)(Q-1)$;
 9 choosing n random numbers e_1 through e_n that are relatively prime to N ;
 10 choosing random numbers e and e' that are relatively prime to N ;
 11 choosing n numbers d_1 through d_n such that $e_i d_i \bmod M$ equals one for
 12 $1 \leq i \leq n$;

13 choosing numbers d and d' such that $ed \bmod M$ is equal to one and such that
 14 $e'd' \bmod M$ is equal to one;

15 generating and storing a database of $\binom{n}{k}$ values, where each value is the
 16 product of d and a unique k of the d_i numbers for $1 \leq i \leq n$;
 17 deleting P , Q , and M ;
 18 computing $S^{ee'}$;
 19 distributing n secret owner pieces to each of the n secret owners, wherein each
 20 of the secret owner pieces includes $S^{ee'}$ and one of the numbers e_1 through e_n ; and
 21 deleting the secret S and e_1 through e_n , e , d_1 through d_n , and d .

1 37. A computer comprising a processor and a computer readable storage
2 medium coupled to the processor having embodied thereon processor readable program code
3 suitable for programming the computer to perform a method performed by a custodian to
4 share a secret among n secret owners such that any k of the n secret owners may reconstruct
5 the secret, the method comprising the steps of:
6 encrypting the secret so as to generate an encrypted secret;
7 deleting the secret; and
8 performing a forward k out of n secret sharing algorithm on the encrypted
9 secret so as to generate n secret owner pieces.

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